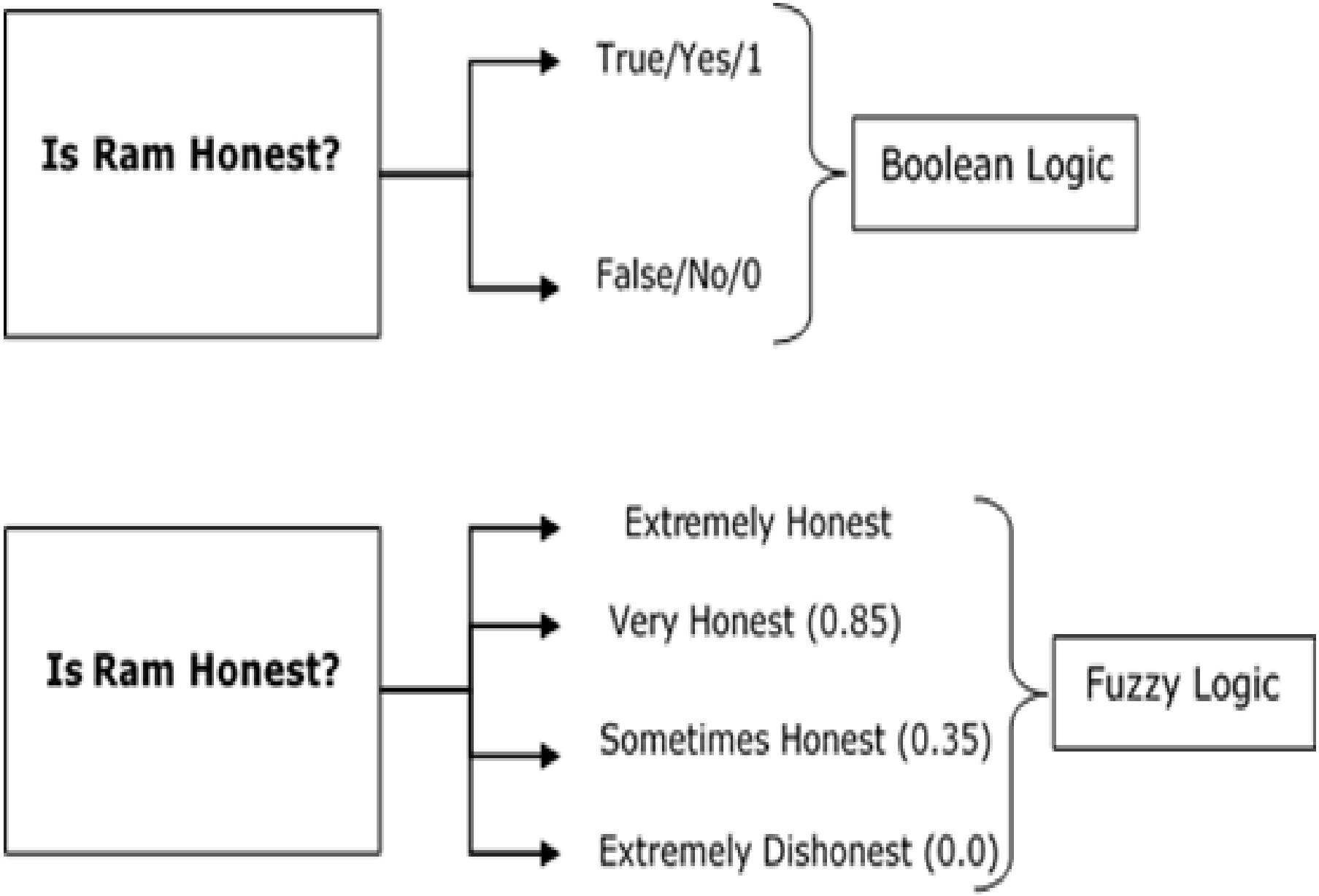
Fuzzy Logic

Fuzzy means VAGUENESS.

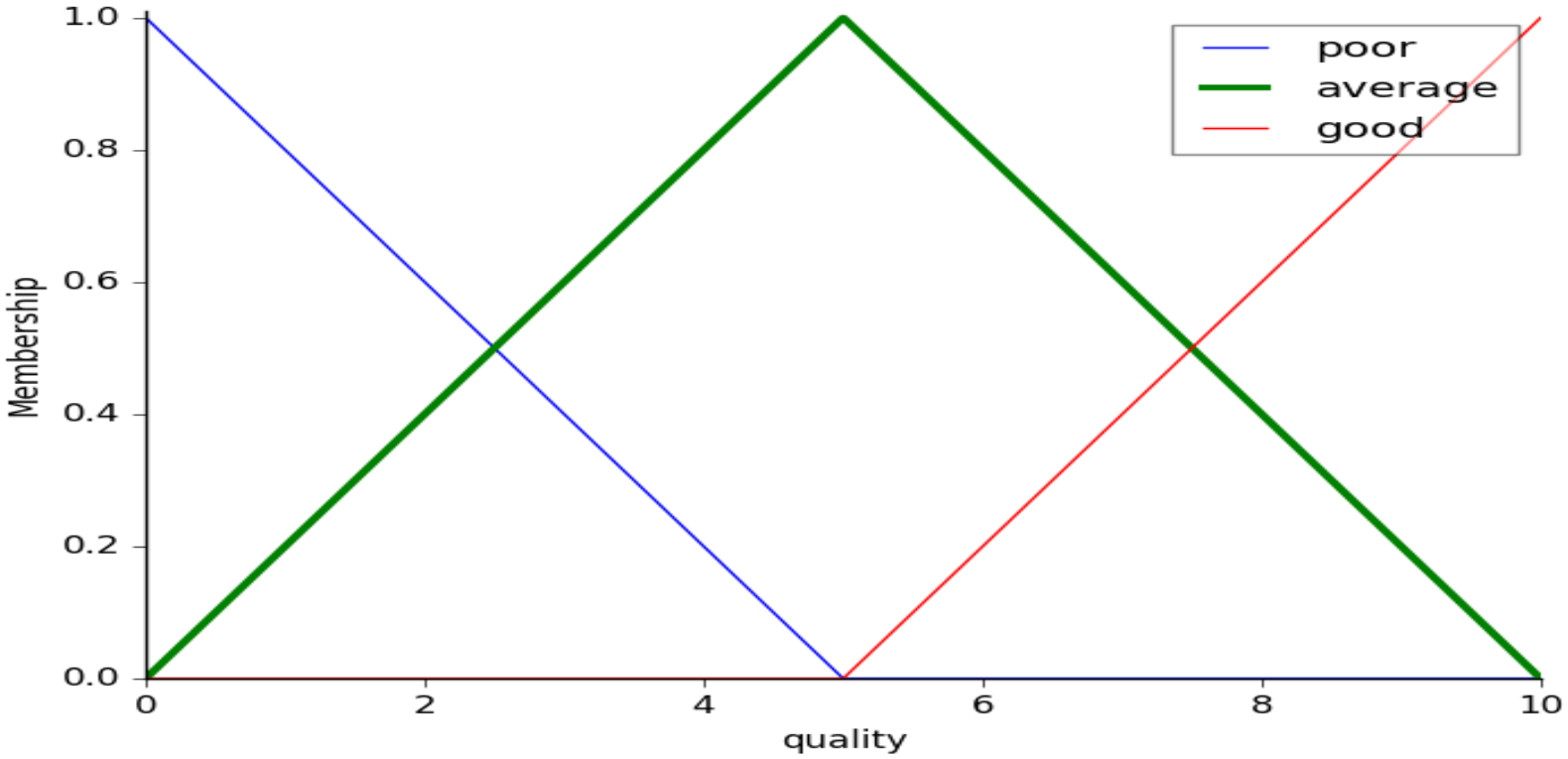
Introduced by Lofti A. Zadeh (1965)

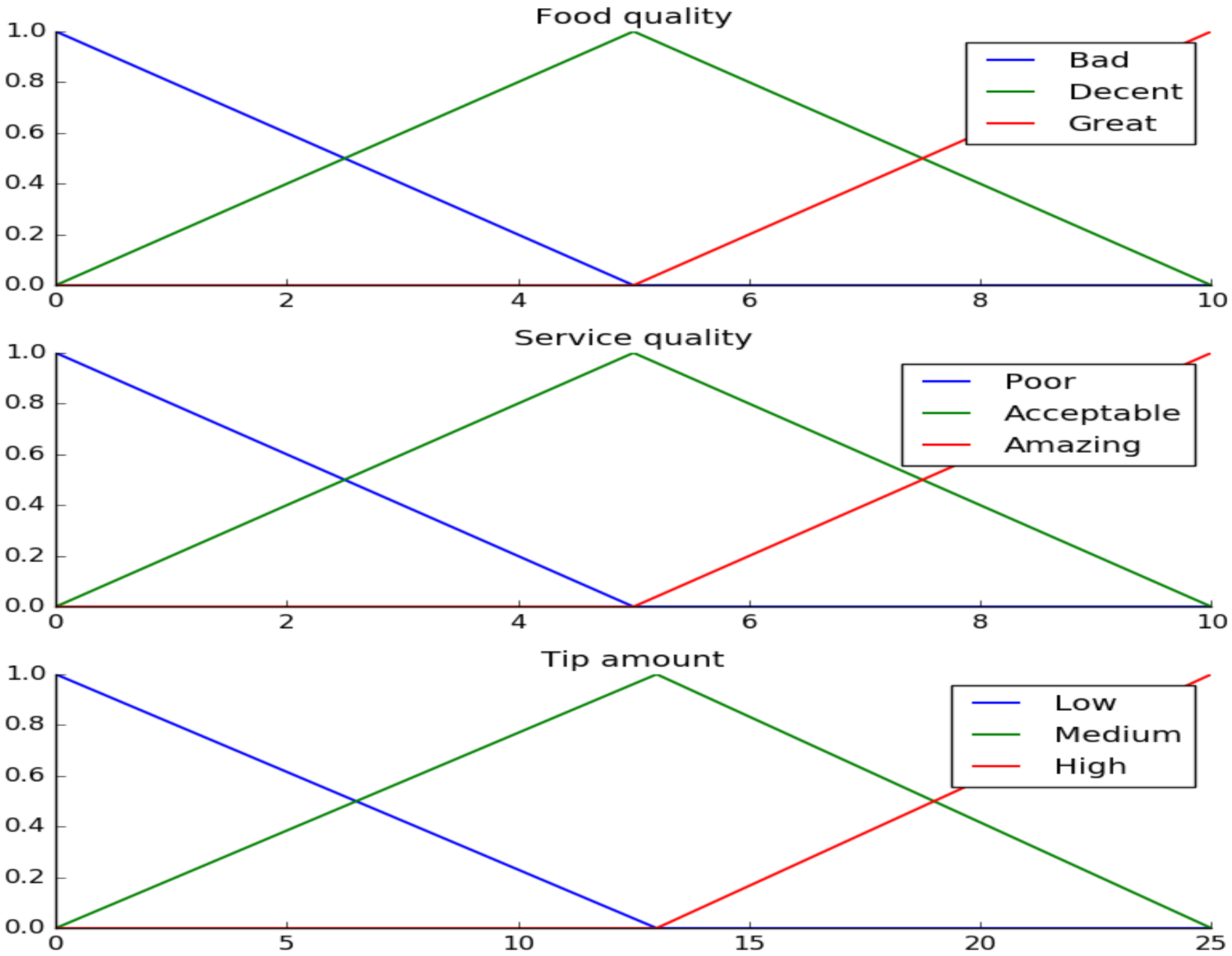

Is Ram Honest?
True/Yes/1
False/No/0
Boolean Logic
Is Ram Honest?
Extremely Honest
Very Honest (0.85)
Sometimes Honest (0.35)
Extremely Dishonest (0.0)
Fuzzy Logic

FUZZY RULES

1. IF the *service* was good *or* the *food quality* was good, THEN the tip will be high.
2. IF the *service* was average, THEN the tip will be medium.
3. IF the *service* was poor *and* the *food quality* was poor THEN the tip will be low.
4. IF the *service* was poor *and* the *food quality* was good THEN the tip will be average.
5. IF the *service* was poor *and* the *food quality* was average THEN the tip will be average.
6. IF the *service* was average *and* the *food quality* was average THEN the tip will be average.

Input 1 (Service) Input 2 (Food Quality) Output (Tip)


1.0
0.8
0.6
0.4
0.2
0.0
Membership
0
2
4
6
8
10
quality
poor
average
good


Food quality
Bad
Decent
Great
Service quality
Poor
Acceptable
Amazing
Tip amount
Low
Medium
High

Example

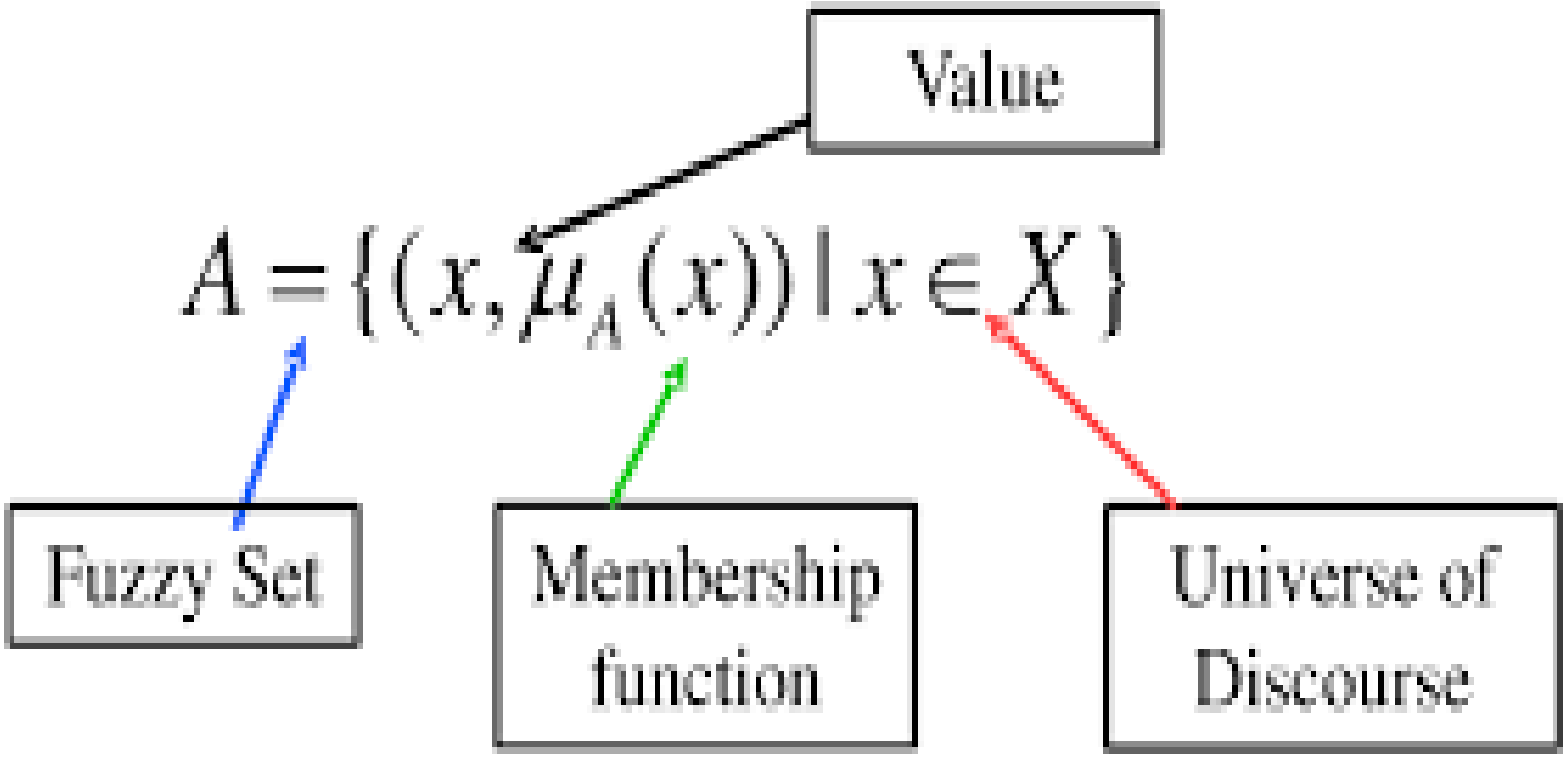
SERVICE

BAD F1 = { (0, 1), (1, 0.8), (2, 0.6), (3, 0.5), (4, 0.4), (5, 0) }

AVERAGE F2 = { }

GOOD F3 = { }

- A fuzzy set can be represented by an ordered set of pairs:



RULE TABLE

FUZZY RELATIONS

$$A = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\} \text{and } B = \{(b_1, 0.5), (b_2, 0.6)\}$$

$$R = A \times B = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

Intersection:

$$A \cap B = \{(x_1,0.5),(x_2,0.2),(x_3,0)\}$$

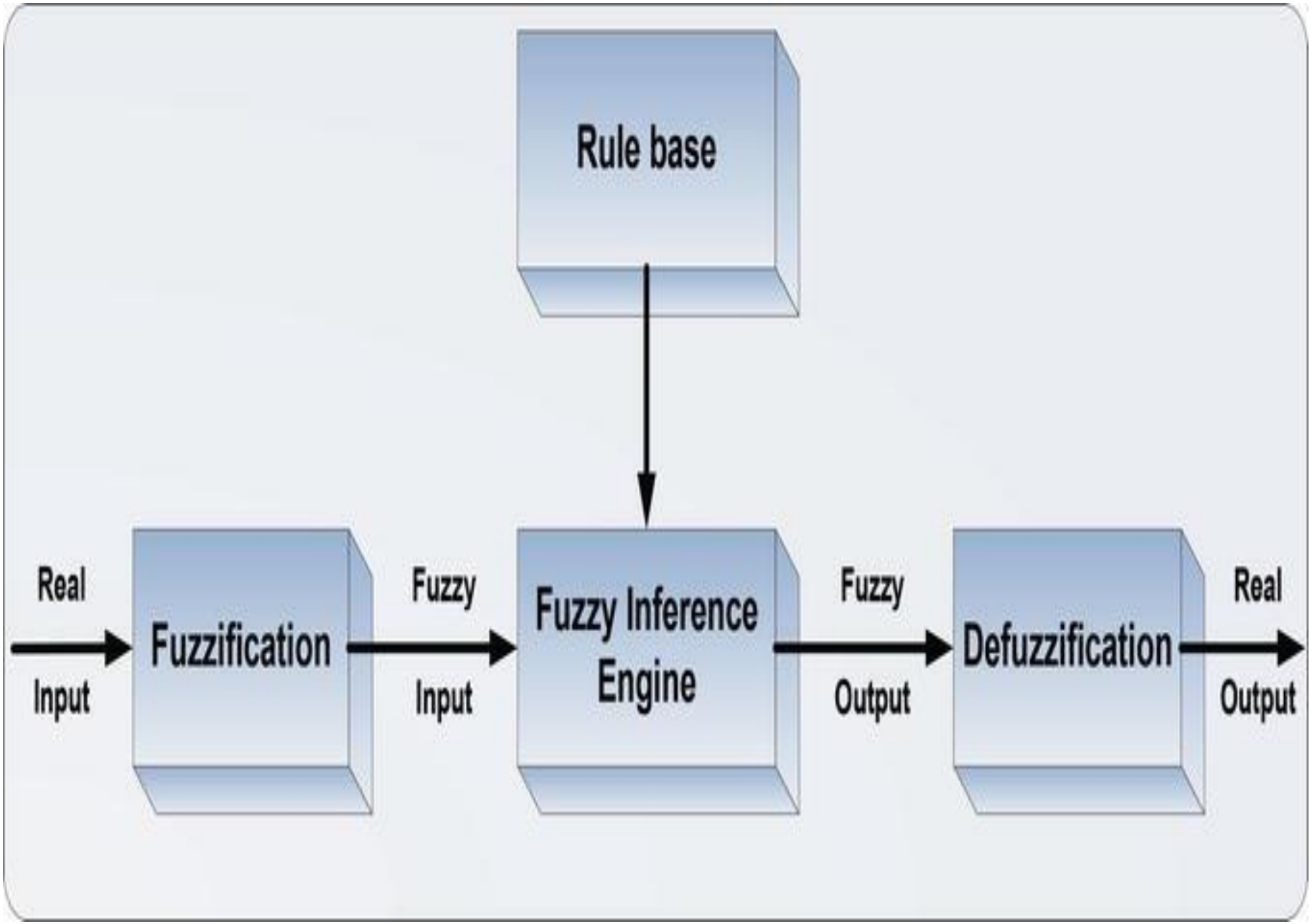
Because

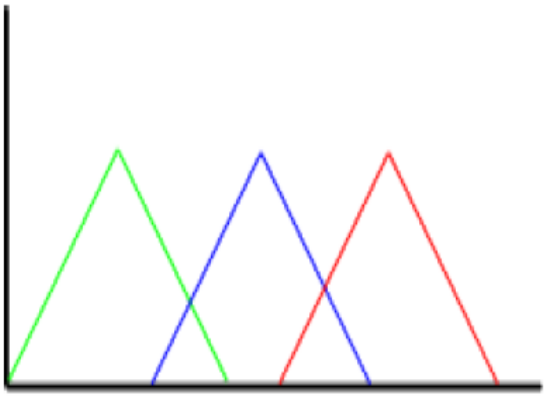
$$\mu_{A\cap B}(x_1) = \min(\mu_A(x_1), \mu_B(x_1))$$

$$= \max(0.5,0.8)$$

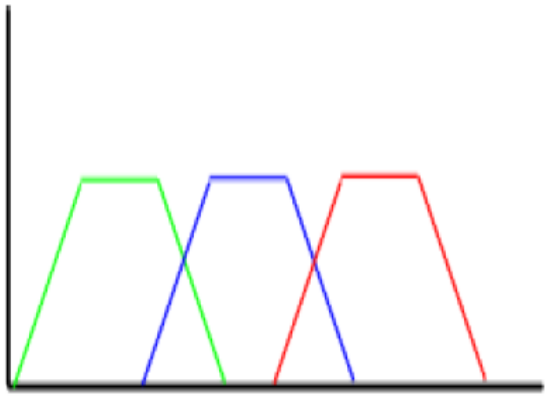
$$= 0.5$$

$$\mu_{A\cap B}(x_2) = 0.2 \text{ and } \mu_{A\cap B}(x_3) = 0$$

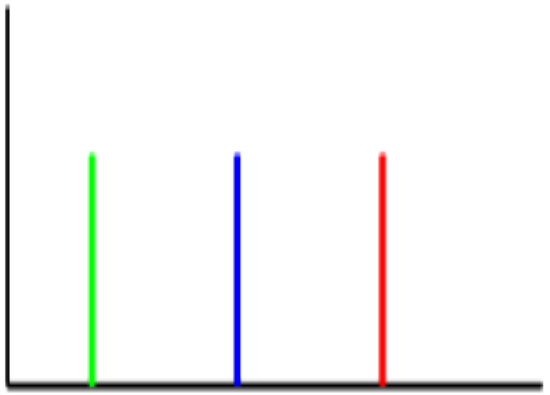

Rule base
Real
Input
Fuzzification
Fuzzy
Input
Fuzzy Inference
Engine
Fuzzy
Output
Defuzzification
Real
Output



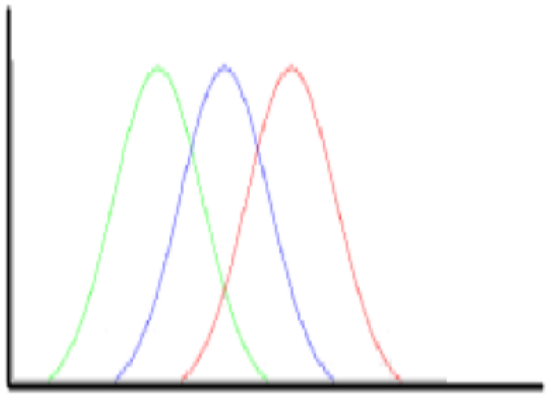
triangular



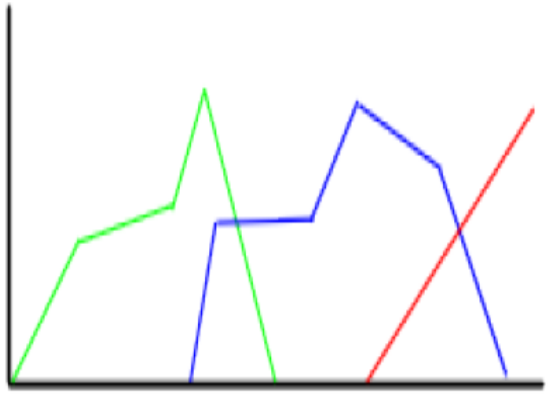
trapezoidal



singleton



Gaussian

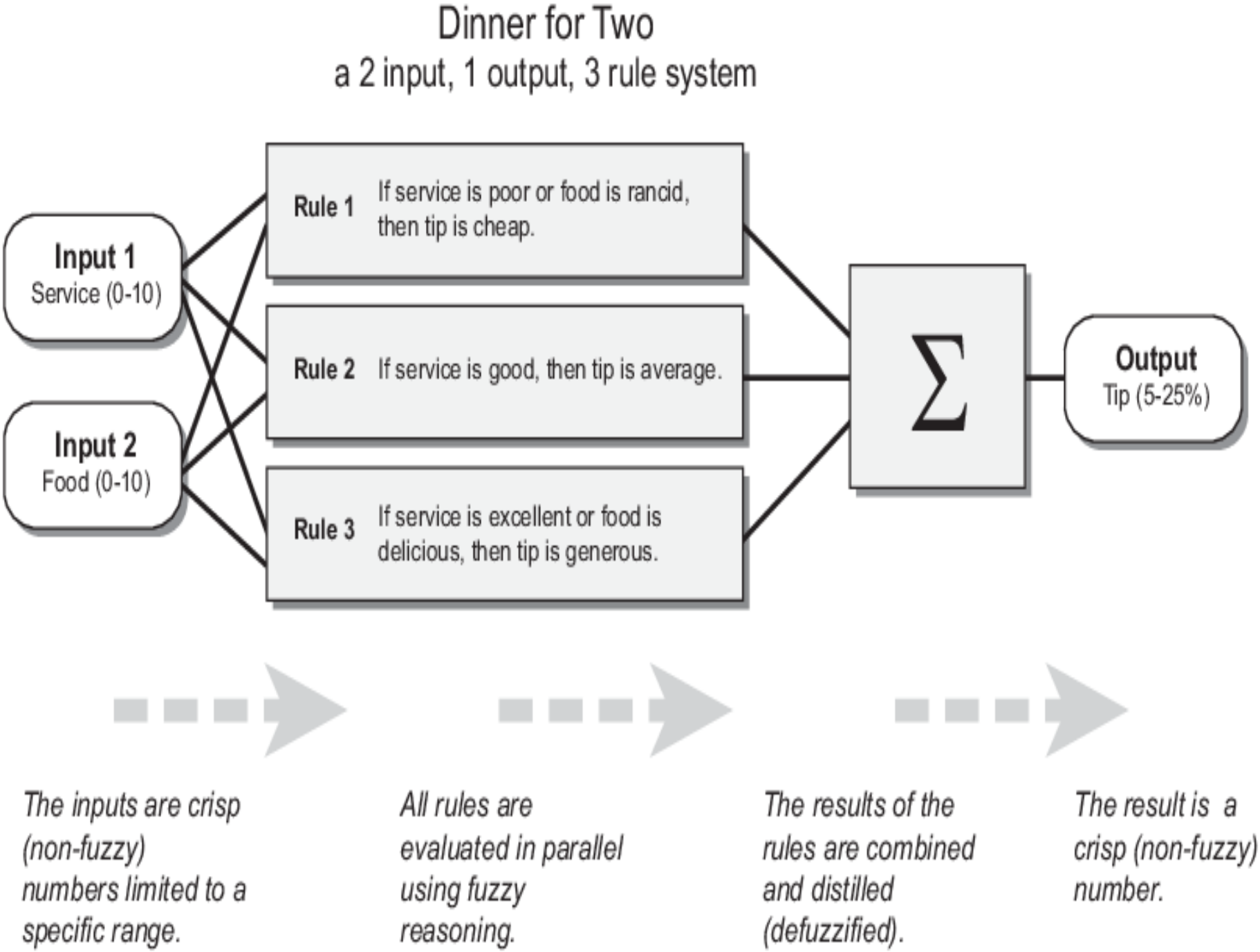


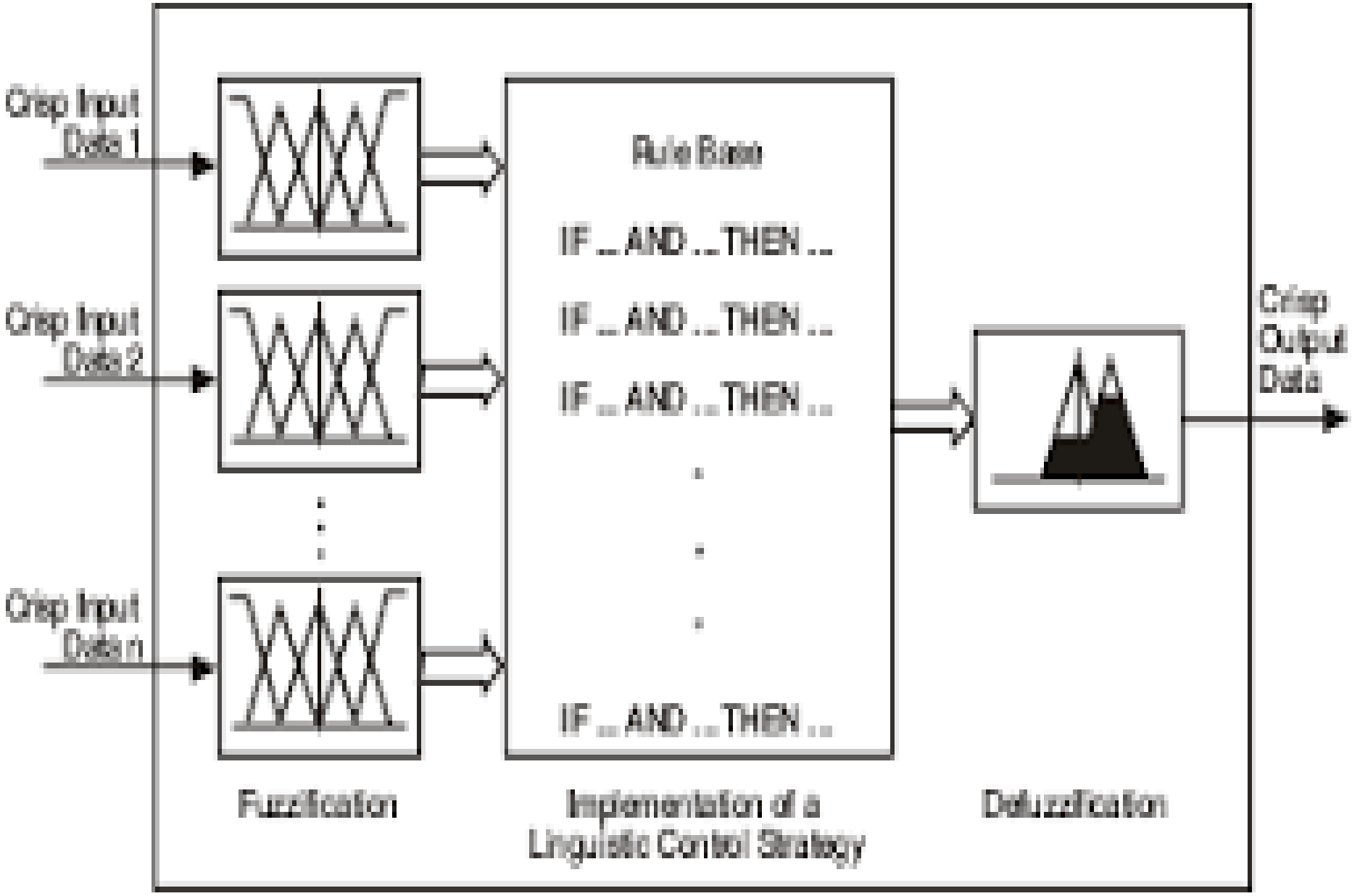
Piecewise linear

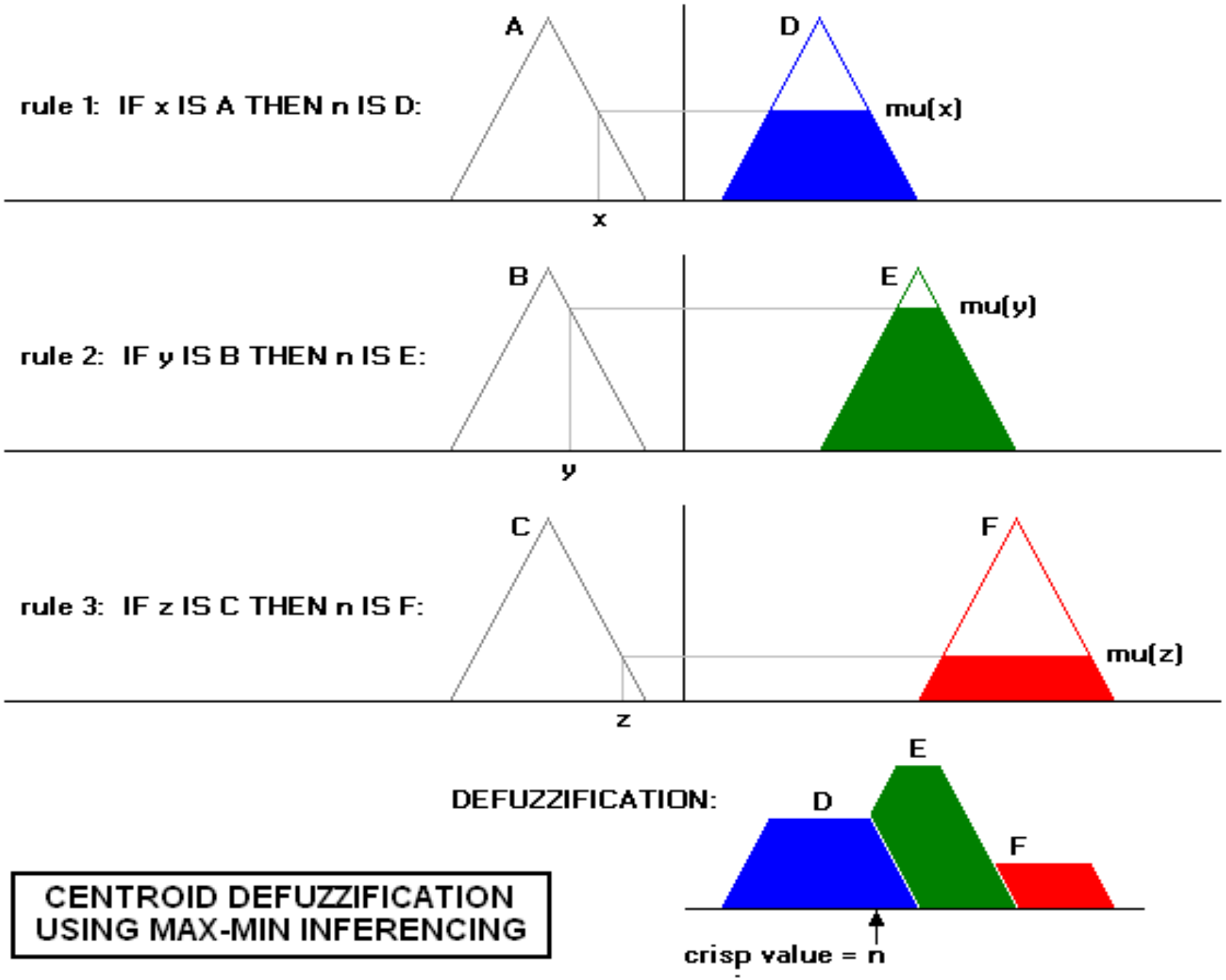
METHODS OF MEMBERSHIP VALUE ASSIGNMENT

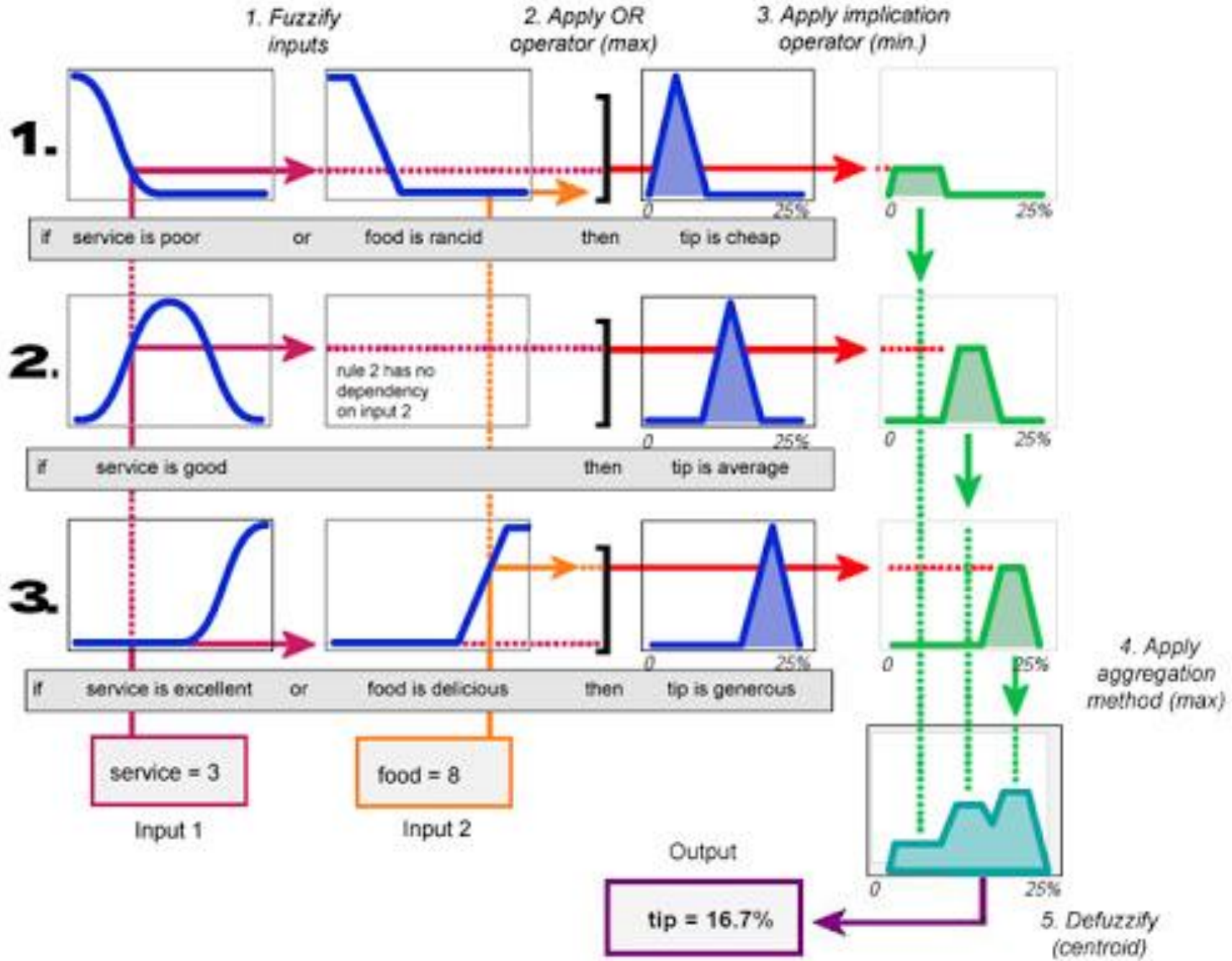
The various methods of assigning membership values are:

- Intuition,
- Inference,
- Rank ordering,
- Angular fuzzy sets,
- Neural networks,
- Genetic algorithm,
- Inductive reasoning.


Dinner for Two
a 2 input, 1 output, 3 rule system
Input 1
Service (0-10)
Input 2
Food (0-10)
Rule 1
If service is poor or food is rancid, then tip is cheap.
Rule 2
If service is good, then tip is average.
Rule 3
If service is excellent or food is delicious, then tip is generous.
Σ
Output
Tip (5-25%)
The inputs are crisp (non-fuzzy) numbers limited to a specific range.
All rules are evaluated in parallel using fuzzy reasoning.
The results of the rules are combined and distilled (defuzzified).
The result is a crisp (non-fuzzy) number.


Crisp Input Data 1
Crisp Input Data 2
Crisp Input Data n
Rule Base
IF ... AND ... THEN ...
IF ... AND ... THEN ...
IF ... AND ... THEN ...
IF ... AND ... THEN ...
Crisp Output Data
Fuzzification
Implementation of a Linguistic Control Strategy
Defuzzification


rule 1: IF x IS A THEN n IS D:
A
D
mu(x)
x
rule 2: IF y IS B THEN n IS E:
B
E
mu(y)
y
rule 3: IF z IS C THEN n IS F:
C
F
mu(z)
z
DEFUZZIFICATION:
E
D
F
crisp value = n
CENTROID DEFUZZIFICATION
USING MAX-MIN INFERENCING


1. Fuzzify inputs
2. Apply OR operator (max)
3. Apply implication operator (min.)
1.
2.
3.
if service is poor or food is rancid then tip is cheap
rule 2 has no dependency on input 2
if service is good then tip is average
if service is excellent or food is delicious then tip is generous
0
25%
4. Apply aggregation method (max)
service = 3
food = 8
Input 1
Input 2
Output
tip = 16.7%
5. Defuzzify (centroid)

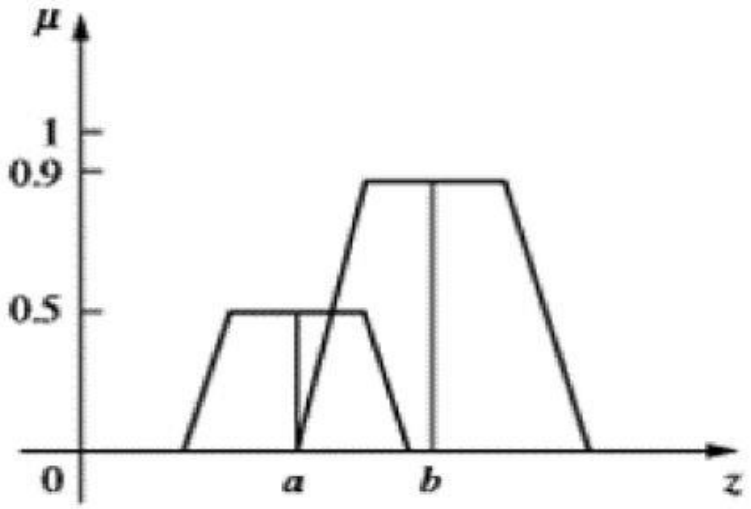
DEFUZZIFICATION METHODS

1. **Lambda-cut method**
2. **Weighted average method**
3. **Maxima methods**
4. **Centroid methods**

Lambda cut Method

Defuzzification

Weighted Average Method



$$z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

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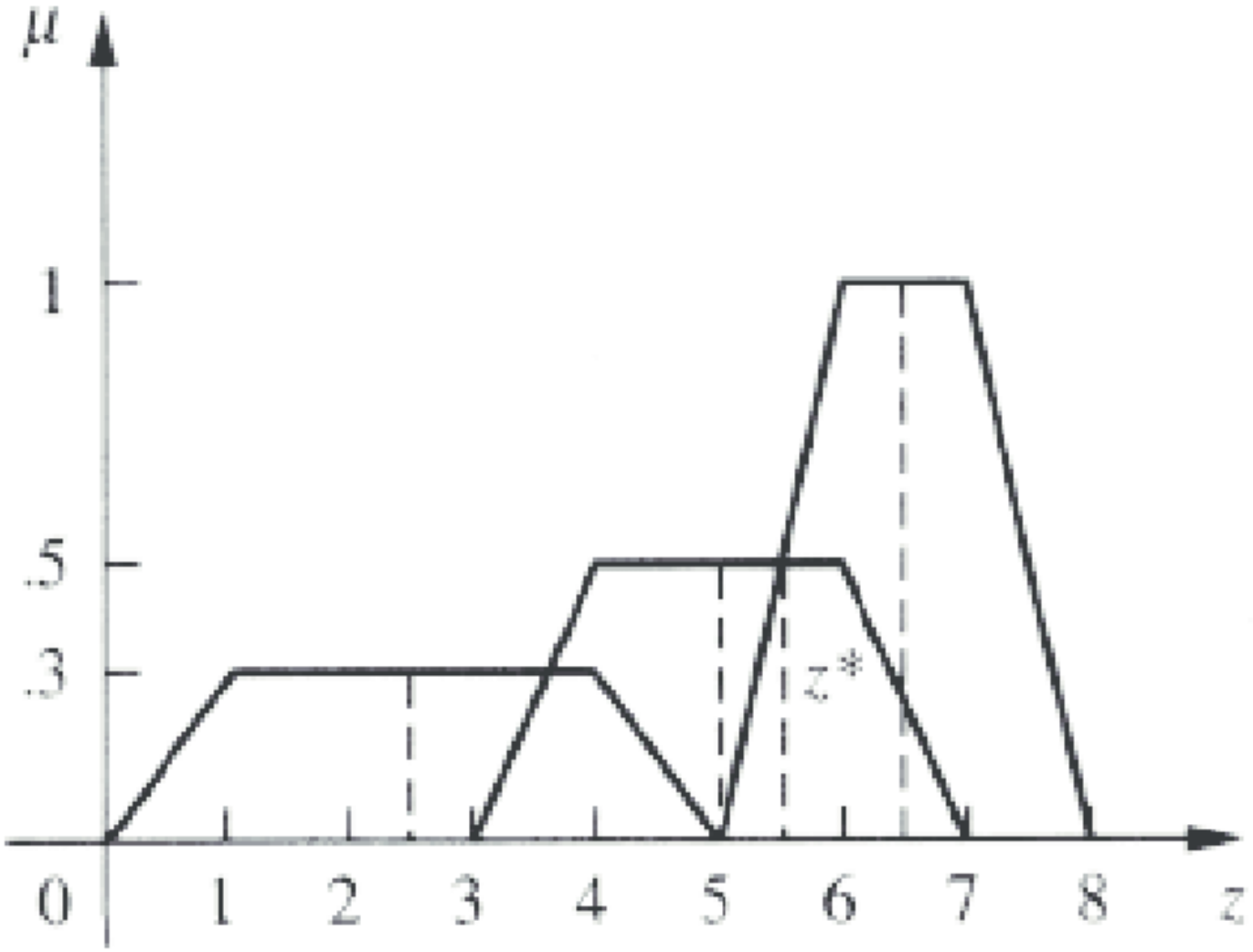
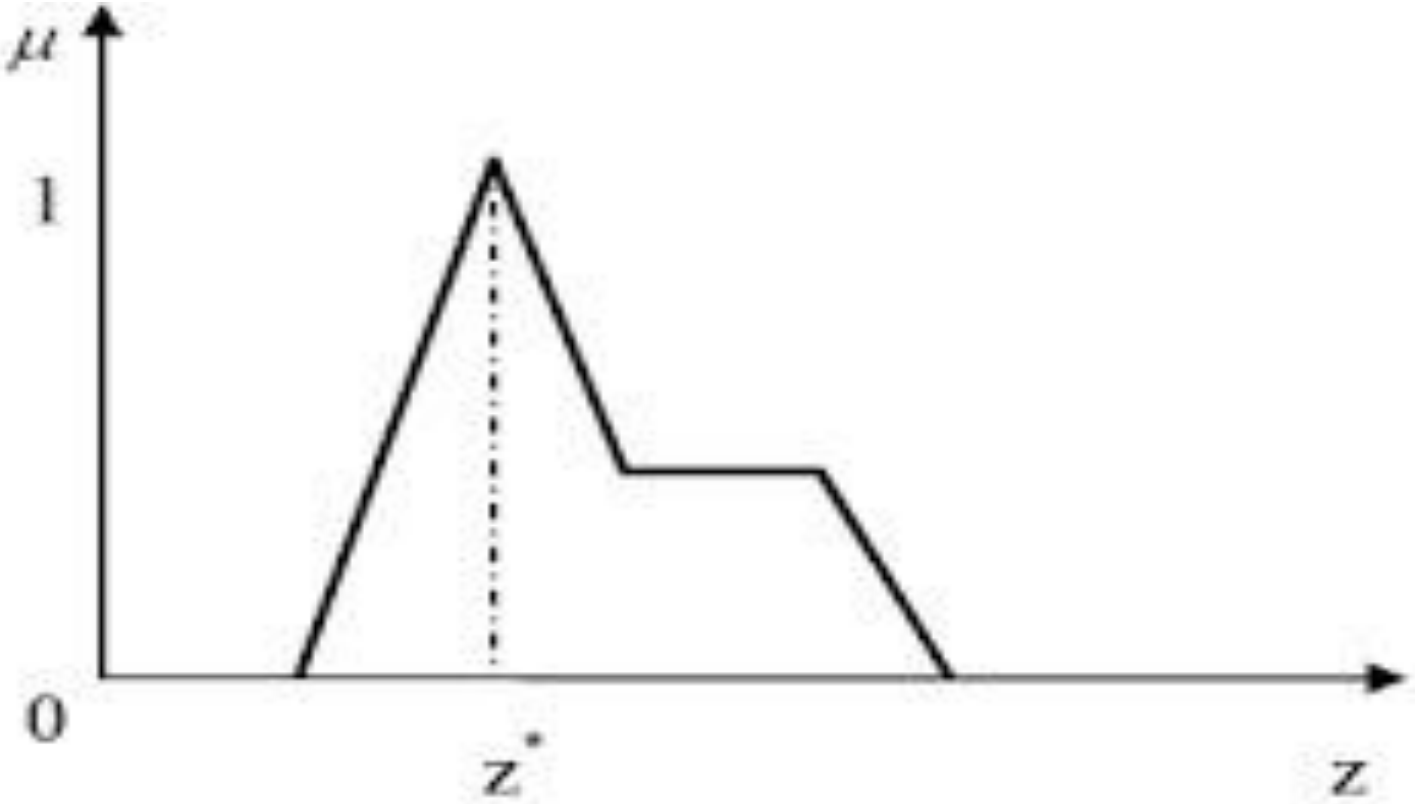
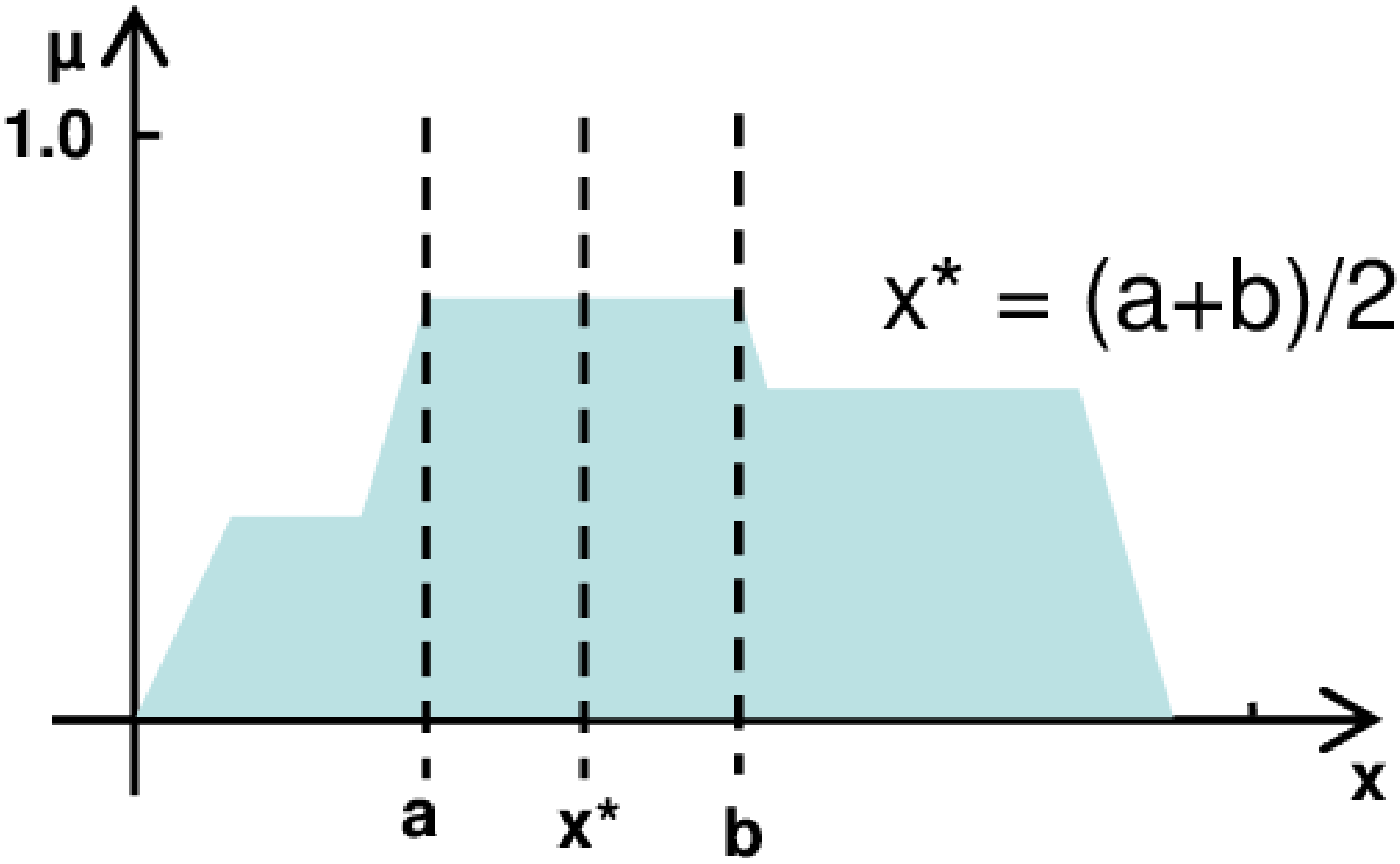


$$z^* = \frac{(.3 \times 2.5) + (.5 \times 5) + (1 \times 6.5)}{.3 + .5 + 1} = 5.41 \text{ meters}$$

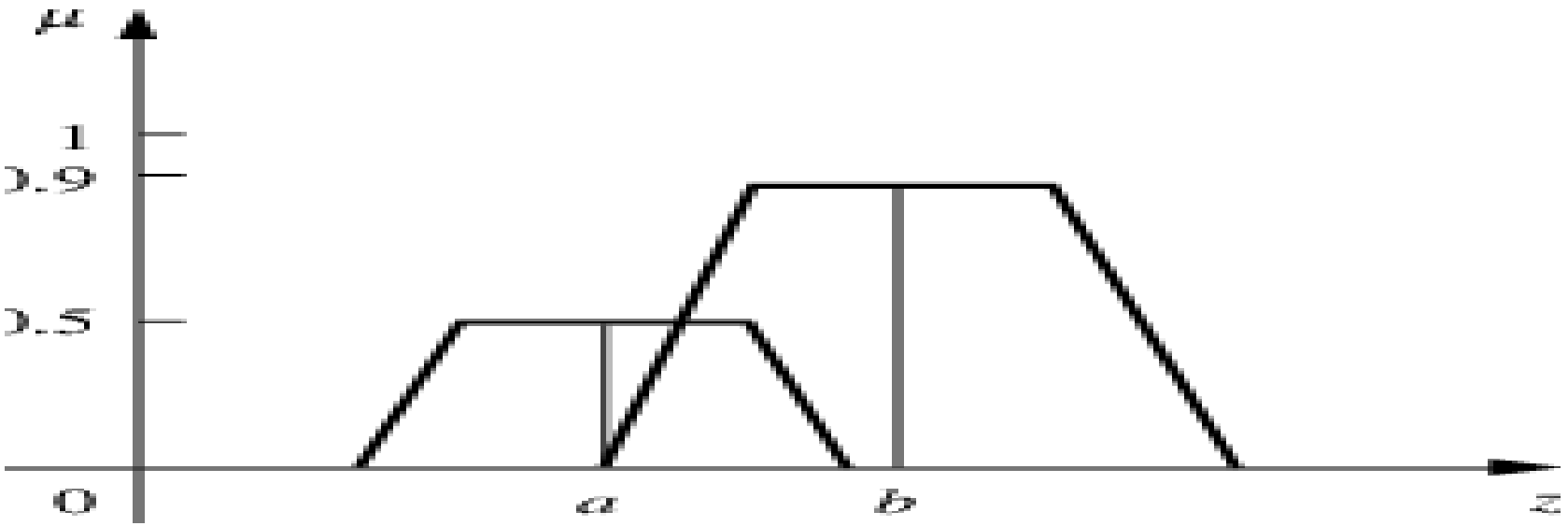
MAXIMA METHOD



MEAN OF MAXIMA



MAXIMA METHODS VS MEAN OF MAXIMA

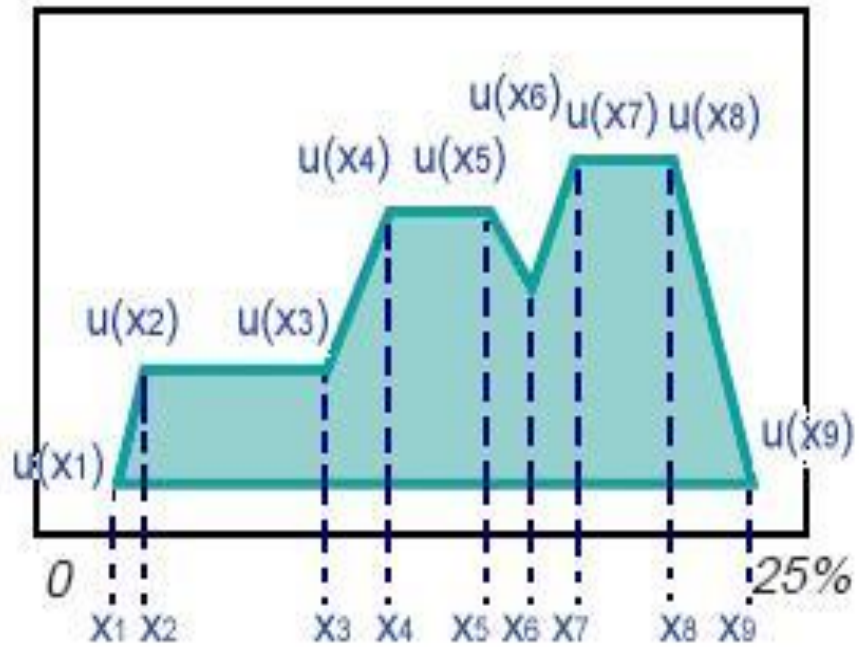


MAXIMA METHOD =➔ b

MEAN OF MAXIMA =➔ (a + b) / 2

center of area method (COA)

5. Defuzzify the aggregate output (centroid)



$$g = \frac{\sum_{i=1}^{9} x_i \cdot u(x_i)}{\sum_{i=1}^{9} u(x_i)} = 16{,}7$$

tip= 16,7%

Result of defuzzification

THANK YOU